# The Gottfried sum rule : theory vs experiment

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## ABSTRACT

The current status of theoretical QCD calculations and experimental measurements of the Gottfried sum rule are discussed. The interesting from our point of view opened problems are summarised. Among them is the task of estimating the measure of light-quark flavour asymmetry in possible future experiments.

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# 1 Introduction.

Studies of the Gottfried sum rule of charged lepton-nucleon deep-inelastic scattering [1], namely

$$I_G(Q^2, 0, 1) = \int_0^1 \frac{dx}{x} \left[ F_2^{lp}(x, Q^2) - F_2^{ln}(x, Q^2) \right] \tag{1}$$

can provide the important information on the possible existence of a light antiquark flavour asymmetry in the nucleon sea. Indeed, the NMC collaboration determination [2] demonstrated that its experimental value

$$I_G^{\text{NMC}}(4 \text{ GeV}^2, 0, 1) = 0.235 \pm 0.026 \quad , \tag{2}$$

is significantly lower than the quark-parton flavour-symmetric prediction

$$I_G(Q^2, 0, 1) = \frac{1}{3} \quad . \tag{3}$$

This deviation is associated with the existence of a non-zero integrated light-quark flavour asymmetry defined as

$$FA(Q^2, 0, 1) = \int_0^1 [\overline{d}(x, Q^2) - \overline{u}(x, Q^2)] dx \quad . \tag{4}$$

In spite of the existence of detailed reviews on the subject [3, 4] we think that it is worth while to return to the consideration of the current status of knowledge of different aspects related to this sum rule.

In this report, based in part on the recent work of Ref. [5], the contributions of QCD corrections and higher-twist effects to the Gottfried sum rule are discussed first in the case of a flavour-symmetric sea. Next, the results of its experimental determination are summarised. Then we briefly outline various possibilities for determinating the integral $FA(Q^2, 0, 1)$ from previos, present and future data.

# 2 QCD predictions.

Let us start by defining the arbitrary non-singlet (NS) Mellin moment of the difference of $F_2$ structure functions of charged lepton-proton and charged lepton-nucleon deep-inelastic scattering (DIS):

$$M_n^{NS}(Q^2) = \int_0^1 x^{n-2} \left[ F_2^{lp}(x, Q^2) - F_2^{ln}(x, Q^2) \right] dx \quad . \tag{5}$$

The moment with $n = 1$, namely the Gottfried sum rule, can be expressed as

$$\begin{aligned} I_G(Q^2, 0, 1) &= \int_0^1 \left[ \frac{1}{3}(u_v(x, Q^2) - d_v(x, Q^2)) + \frac{2}{3}(\overline{u}(x, Q^2) - \overline{d}(x, Q^2)) \right] dx \\ &= \frac{1}{3} - \frac{2}{3} FA(Q^2, 0, 1) \end{aligned} \tag{6}$$

where $u_v(x, Q^2) = u(x, Q^2) - \overline{u}(x, Q^2)$ and $d_v(x, Q^2) = d(x, Q^2) - \overline{d}(x, Q^2)$ are the valence-quark distributions and the measure of flavour asymmetry is related to the difference of the sea-quark distributions $\overline{u}(x, Q^2)$ and $\overline{d}(x, Q^2)$ via Eq.(4).

## 2.1 Perturbative contributions.

Consider first the case when the sea is flavour-symmetric. In zeroth order of perturbation theory the quark-parton result of Eq.(3) is reproduced. However, the quark-gluon interactions generate non-zero corrections to $I_G$, defined as

$$I_G(Q^2, 0, 1) = AD(\alpha_s)C(\alpha_s) \quad . \tag{7}$$

The anomalous dimension term is related to the anomalous dimension function of the first moment $\gamma^{n=1}(\alpha_s)$ and to the QCD $\beta$-function, namely :

$$
\begin{aligned}
AD(\alpha_s) \;=\; & exp\left[ - \int_\delta^{\alpha_s(Q^2)} \frac{\gamma^{n=1}(x)}{\beta(x)} dx \right] = 1 + \frac{1}{2}\frac{\gamma_1^{n=1}}{\beta_0}\left(\frac{\alpha_s(Q^2)}{4\pi}\right) \\
& + \frac{1}{4}\left( \frac{1}{2}\frac{(\gamma_1^{n=1})^2}{\beta_0^2} - \frac{\gamma_1^{n=1}\beta_1}{\beta_0^2} + \frac{\gamma_2^{n=1}}{\beta_0}\right)\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2.
\end{aligned}
\tag{8}
$$

The given expansion in $\alpha_s$ can be obtained after taking into account that $\gamma_0^{n=1} = 0$ and setting $\delta = 0$.

The calculations of $\gamma_1^{n=1}$ [6, 7] give the following result

$$\gamma_1^{n=1} = -4(C_F^2 - C_F C_A)[13 + 8\zeta(3) - 2\pi^2] = +2.557 \quad . \tag{9}$$

The coefficients of the QCD $\beta$-function are well-known:

$$\beta_0 \;=\; \left(\frac{11}{3}C_A - \frac{2}{3}f\right) = 11 - 0.667f \tag{10}$$

$$\beta_1 \;=\; \left(\frac{34}{3}C_A^2 - 2C_F f - \frac{10}{3}C_A f\right) = 102 - 12.667f \quad . \tag{11}$$

Here and below $C_F = 4/3$ and $C_A = 3$ and $f$ is the number of active flavours.

The general order $\alpha_s^2$- expression for $C(\alpha_s)$ can be written down as

$$C(\alpha_s) = \frac{1}{3}\left[1 + C_1^{n=1}\left(\frac{\alpha_s}{\pi}\right) + C_2^{n=1}\left(\frac{\alpha_s}{\pi}\right)^2\right] \tag{12}$$

where $C_1^{n=1} = 0$ [8]. The coefficient $C_2^{n=1}$ was evaluated only recently [5] by means of numerical integration of the complicated $x$-dependence of the two-loop contributions to the coefficient functions of DGLAP equation for DIS structure functions, calculated in Ref. [9]. The expression obtained in Ref. [5] is:

$$C_2^{n=1} = (3.695C_F^2 - 1.847C_F C_A) = -0.821 \quad . \tag{13}$$

Collecting now all known QCD corrections to Eq.(7) we found [5]:

$$I_G(Q^2, 0, 1)_{f=3} = \frac{1}{3}\left[1 + 0.0355\left(\frac{\alpha_s}{\pi}\right) + \left(-0.862 + \frac{\gamma_2^{n=1}}{64\beta_0}\right)\left(\frac{\alpha_s}{\pi}\right)^2\right] \quad , \tag{14}$$

$$I_G(Q^2, 0, 1)_{f=4} = \frac{1}{3}\left[1 + 0.0384\left(\frac{\alpha_s}{\pi}\right) + \left(-0.809 + \frac{\gamma_2^{n=1}}{64\beta_0}\right)\left(\frac{\alpha_s}{\pi}\right)^2\right] \quad , \tag{15}$$

where $\alpha_s = \alpha_s(Q^2)$ and the three-loop anomalous dimension term $\gamma_2^{n=1}$ is still unknown. In Ref.[5] it was estimated using the feature observed in Ref.[12] that the $n$-dependence of the ratio $\gamma_1^n/\gamma_2^n$, obtained from three-loop terms of the the anomalous dimension functions of even moments for charged lepton- nucleon DIS, calculated in Ref.[10], and of odd moments of $\nu N$ DIS, calculated in Ref. [11], can be fixed by similar approximate relation. Taking these estimates into account we got [5]

$$I_G(Q^2,0,1)_{f=3} = \frac{1}{3}\left[1 + 0.0355\left(\frac{\alpha_s}{\pi}\right) - 0.811\left(\frac{\alpha_s}{\pi}\right)^2\right] \quad , \tag{16}$$

$$I_G(Q^2,0,1)_{f=3} = \frac{1}{3}\left[1 + 0.0384\left(\frac{\alpha_s}{\pi}\right) - 0.822\left(\frac{\alpha_s}{\pi}\right)^2\right] \quad , \tag{17}$$

where the $\alpha_s^2$ contribution is dominated by the numerical value of the coefficient $C_2^{n=1}$ from Eq. (13). Thus we convinced ourselves that the perturbative QCD corrections to the Gottfried sum rule are really small and cannot be responsible for violation of the flavour-symmetric prediction from the experimental value of Eq.(2).

## 2.2 Higher-twist terms.

The possibility that the higher-twist effects in the Gottfried sum rule might be sizeable was discussed in Ref. [13]. It was argued that the next-to-leading sets of parton distributions, namely GRV94 [14], MRST98 [15] and CTEQ5 [16], failed to describe the existing experimental $F_2^p$-$F_2^n$ data below $Q^2 < 7$ GeV$^2$ [13]. From the point of view of the authors of Ref. [13] this might be associated with substantial higher-twist corrections, which in part are responsible for the deviation of the Gottfried sum rule result from its NMC value. However, definite results of the fits to $F_2^p - F_2^n$ data, performed in Ref.[17] with the help of the most recent Alekhin PDF set of Ref. [18] (A02), indicate that that the conclusions of Ref. [13] are too optimistic. Indeed, the authors of Ref. [17] demonstrated that for the second NS moment of Eq. (5) the numerator of the $1/Q^2$ twist-4 correction is rather small, namely $H_{F_2}^{n-p} = -0.0058 \pm 0.0069$ GeV$^2$. In view of this we expect that the twist-4 contribution to the first moment, namely to $I_G$, will be small also. Moreover, in spite of the fact that the fits of Ref. [17] reveal a definite discrepancy between $x$-dependence found for $H_{F_2}^{n-p}(x)$ and the predictions of the infrared renormalon (IRR) model (for a review see Ref. [19]), we think that the latter method still might give order-of-magnitude estimates of the higher-twist contributions. Note, that the IRR model is based in part on summations of the chain of fermion loop insertions to the gluon propagator and is thus related to the large $f$-expansion of the coefficient functions. For the polarised Bjorken sum rule these studies were made in Ref.[20] (for more recent discussions see Ref.[21]). In the case of $I_G$ the flavour dependence does not manifest itself up to $\alpha_s^3$-corrections (see Eq. (12) ). Thus, we conclude, that in comparison with polarised Bjorken sum rule, the IRR model corrections (and therefore higher-twist effects) to the Gottfried sum rule will be damped by the additional factor $\alpha_s/\pi$. In view of this we think that the Gottfried sum rule cannot receive substantial higher twist contributions, though the explicit demonstration of the validity of this statement is still missing.

# 3    Experimental situation.

The experimental determinations of the Gottfried sum rule have a rather long history, summarised in the reviews of Refs.[3, 4]. In fact what is really evaluated from the experimental data is the integral

$$I(Q^2, x_{min}, x_{max}) = \int_{x_{min}}^{x_{max}} \frac{dx}{x} \left[ F_2^{lp}(x, Q^2) - F_2^{ln}(x, Q^2) \right] \quad . \tag{18}$$

In a more detailed analysis $F_2^{lp}(x, Q^2) - F_2^{ln}(x, Q^2)$ should be extrapolated to the unmeasured regions and since $F_2^{ln}(x, Q^2)$ is extracted from DIS on nuclei targets, nuclear effects should be also taken into account. However, all four experimental groups working on the direct determiantion of the Gottfried sum rule from their experimental data, were not able to achieve ideal results. Indeed, the main source of experimental uncertainty results from the extrapolations of the experimental data from $x_{min}$ to 0. Moreover, the mean $Q^2$ in the data are dependent on $x$ and it is sometimes difficult to fix typical $Q^2$ value of the Gottfried sum rule (see Table 1.)

| Group | $Q^2$ (GeV$^2$) | $x_{min}$ | $x_{max}$ | $I_G(Q^2, x_{min}, x_{max})$ |
|---|---|---|---|---|
| SLAC [22] | 0.1–20 | 0.02 | 0.82 | 0.200±0.040 |
| EMC [23] | 10–90 | 0.02 | 0.8 | 0.197±0.011(st.)±0.083 (sys.) |
| BCDMS [24] | 20 | 0.06 | 0.8 | 0.197±0.011 (st.)±0.036(sys.) |
| NMC [2] | 4 | 0.004 | 0.8 | 0.221±0.008 (st.)±0.019(sys.) |

**Table 1.** The existing experimental data for the integral of Eq. (18).

In spite of the fact that already the results of Ref. [22] inspired discussions of the possibility that the theoretical expression for Eq.(3) might be violated, its further determinations by EMC collaboration [23], namely

$$I_G^{\text{EMC}}(Q^2 =?, 0, 1) = 0.235_{-0.099}^{+0.110} \tag{19}$$

within experimental error bars did not in fact demonstrate an obvious deviation from the quark-parton model prediction 1/3 (note, that in Eq.(19) $Q^2$-value was not determined). A similar conclusion also applies to the analysis of the BCDMS data in Ref. [24]. Indeed, it suffers from large uncertainties at $x < 0.06$, which vary from 0.07 to 0.22. Thus the results of NMC collaboration of Eq.(2) turned out to be extremely important for understanding that flavour-asymmetry of antiquark distributions in the nucleon really exist in nature. The precision of their data even allow one to extract the value of integrated light-quark flavour asymmetry, defined by Eq.(4) [2]:

$$FA^{\text{NMC}}(4 \text{ GeV}^2, 0, 1) = 0.147 \pm 0.039 \quad . \tag{20}$$

However, even the members of NMC collaboration were not able to take into account all effects, typical for DIS. Indeed, the nuclear corrections, such as the Fermi motion, were neglected by them. In view of this, it became rather important to get an independent

experimental extraction of $FA(Q^2, 0, 1)$. Quite recently this was done in an analysis of the data for Drell-Yan production in proton-proton and proton-deuteron scattering by the members of E866 collaboration. Integrating $\overline{d}(x, Q^2) - \overline{u}(x, Q^2)$ over the measured $x$-region they obtained [25]:

$$FA^{\mathrm{E866}}(54 \text{ GeV}^2, 0.015, 0.35) = 0.0803 \pm 0.011 \quad . \tag{21}$$

Extrapolation this integral to the unmeasured region $0 \le x \le 0.015$ and assuming that the contribution for $x \ge 0.35$ is negligible, the members of E866 collaboration found that

$$FA^{\mathrm{E866}}(54 \text{ GeV}^2, 0, 1) = 0.118 \pm 0.015 \tag{22}$$

(see Ref.[25]). Within existing error-bars this result turned out to be in agreement with Eq. (20), which is the NMC value of this integral, extracted at 4 GeV$^2$. In view of this it is possible to conclude that the value of the integrated light-quark flavour asymmetry is almost independent of $Q^2$ over a wide range of the momentum transfer. This demonstrates in part its non-perturbative origin.

# 4   Possible future prospects.

In order to calculate the characteristics of a light-quark flavour asymmetry a number of non-perturbative models have been successfully used. Among them are the meson-cloud model, instanton model, chiral-quark soliton models and others (for reviews see Refs. [3, 4]). However, to make more definite conclusions on the predictive power of these approaches more detailed experimental knowledge about the behaviour of the rate $\overline{d}/\overline{u}$ at different values of $x$ is needed.

The new 120 GeV Fermilab Main Injector should allow one to extend Drell-Yan measurements of $\overline{d}/\overline{u}$ to the region of $0.02 < x < 0.7$. Moreover, the studies of the CEBAF data for $F_2^D$ al large $x$ can give the chance to perform more detailed combined fits of all available DIS data. The extraction of a light-quark flavour asymmetry from the fits to the data for $F_2^D$ require the detailed treatment of nuclear effects, say in the manner of the work of Ref.[17]. Clearly, these measurements might be important for more detailed determinations of the effects of flavour-asymmetry in various sets of parton distribution functions, which at present differ in the CITEQ6M, MRST2001C and A02 sets (for their comparison at $Q^2 = (100 \text{ GeV})^2$ see Ref. [26]). As a test of their current predictive power it can be rather useful to use them for calculations of the integral $FA(Q^2, 0, 1)$, as was done in Ref.[4].

Another possibility is to study light-quark flavour asymmetry in the process of $\nu N$ DIS at the possible future Neutrino Factories. Indeed, as was noticed in Ref. [6] in the quark-parton model the $\overline{d} - \overline{u}$ difference can be related to the $\nu N$ DIS SFs as

$$\overline{d}(x) - \overline{u}(x) = \frac{1}{2}\left[F_1^{\overline{\nu}p}(x) - F_1^{\nu p}(x)\right] - \frac{1}{4}\left[F_3^{\overline{\nu}p}(x) - F_3^{\nu p}(x)\right] \tag{23}$$

where the $\overline{s}$ and $\overline{c}$ distributions are neglected. At the NLO of perturbative QCD the analog of Eq.(23) was also derived [6]. Since it is known, that at the Neutrino Factories it will

be possible to extract from the cross-sections of $\nu N$ DIS $F_1$ and $F_3$ structure functions separately [27], the more precise $\nu N$ DIS data might be useful for additional estimates of the size of the light-quark flavour asymmetry.

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